Editor's Comments, continued from page 2.

be included in the December and subsequent issues. However, I have included a few personal comments in an article in this issue.

To those starting a new school year, my best wishes for a fine one. To all, may the next months be prosperous, productive, and satisfying.

Ross

President's Message, continued from page 3.

workshops run by a chapter, so long as the chapter has already made use of all support mechanisms provided by the Section. In that case, up to $500/chapter is available by applying to Jay.

As successful as the above two programs have been, I believe that the programs are not fully understood or appreciated. Consequently, I was delighted when Jay authored a new handbook for new chapter officers, which spells out how chapters can obtain support from our Society. I have asked Jay to distribute the new manuscript widely so that everyone interested in chapter activities will know what services are available. Also, I hope that AP-S members will react to this document by suggesting new ways by which AdCom can support chapter activities.

If you would like a copy of Jay's handbook, please write to him at:

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AdCom is very interested in improving our support of AP-S Chapters.

Introducing Robert S. Elliott
Feature Article Author

Robert S. Elliott has been a Professor of Electrical Engineering at UCLA since 1957. He holds an AB in English Literature and a BS in Electrical Engineering from Columbia University ('42, '43), an MS and PhD in Electrical Engineering from the University of Illinois ('47, '52), and an MA in Economics from the University of California at Santa Barbara ('71).

His prior experience includes periods at the Applied Physics Laboratory of John Hopkins and at the Hughes Research Laboratories, where he headed the antenna research activities. Dr. Elliott has also been on the faculty of the University of Illinois and was a founder of Rantec Corporation, serving as its first Vice President and Technical Director. He is presently serving a second two-year stint as a Distinguished Lecturer for the IEEE and was Chairman of the Coordinating Committee for the 1981 IEEE AP-S International Symposium, held in Los Angeles. He currently serves as a consultant to Hughes, Canoga Park. In addition to being a Fellow of the IEEE, Dr. Elliott is also a member of Sigma Xi, Tau Beta Pi, and the New York Academy of Sciences. He has received numerious teaching prizes and is the recipient of two Best Paper awards by the IEEE.

Feature Articles Solicited for Newsletter

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The editorial Staff of the *AP-S Newsletter* continues to actively solicit feature articles which describe engineering activities taking place in industry, government, and universities. Emphasis is being placed on providing the reader with a general understanding of the technical problems being addressed by various engineering organizations as well as their capabilities to cope with these problems. If you or anyone else in your organization is interested in submitting an article, we encourage you to contact Editor Ross Stone to discuss the appropriateness of the topic. He may be reached at the above address.

Feature Article

Array Pattern Synthesis

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Summary -- This pair of review articles describes various synthesis techniques which have proven useful in determining the excitations of arrays that are to produce desired radiation patterns. Part I deals with equispaced linear arrays and Part II with planar arrays. The latter may have its radiating elements disposed in rectangular, triangular, or circular grids, and the boundary can be arbitrary, e.g., rectangular, circular, or elliptical. Sum and difference patterns (characterized by lobes interspersed by deep nulls) and shaped patterns (such as cosecant-squared) are discussed. Only the array factor is considered; the resulting patterns need to be multiplied by the element factor.

Part I: Equispaced Linear Arrays

For a linear array of N+1 elements, laid out along the Z axis, with an interelement spacing d, the array factor is

$$A(\theta) = \sum_{n=0}^{N} I_n e^{jnkd\cos\theta} \qquad (1)$$

with I_n the excitation of the nth radiating element and $k=2\pi/\lambda$ the wave number; θ is the angle measured with respect to the zenith, i.e., the positive direction along the Z axis. When one makes the substitutions

$$\psi = kd\cos\theta \qquad w = e^{j\psi} \qquad (2)$$

the array factor becomes $A(w) = I_N f(w)$ where

$$f(w) = \sum_{n=0}^{N} (I_n/I_N)w^n = \prod_{n=1}^{N} (w-w_n) \qquad (3)$$

Since f(w) differs from $A(\theta)$ only by a multiplicative constant, it too can be viewed as the array factor.

Schelkunoff [1], realizing the implications of equation (3), introduced the extremely useful idea of constructing a unit circle in the complex w plane. The variable w, which is a surrogate for θ, is constrained to move along this circle. The array factor f(w) is thus controlled by placement of the roots w_n. One reaches the important conclusion that the synthesis of

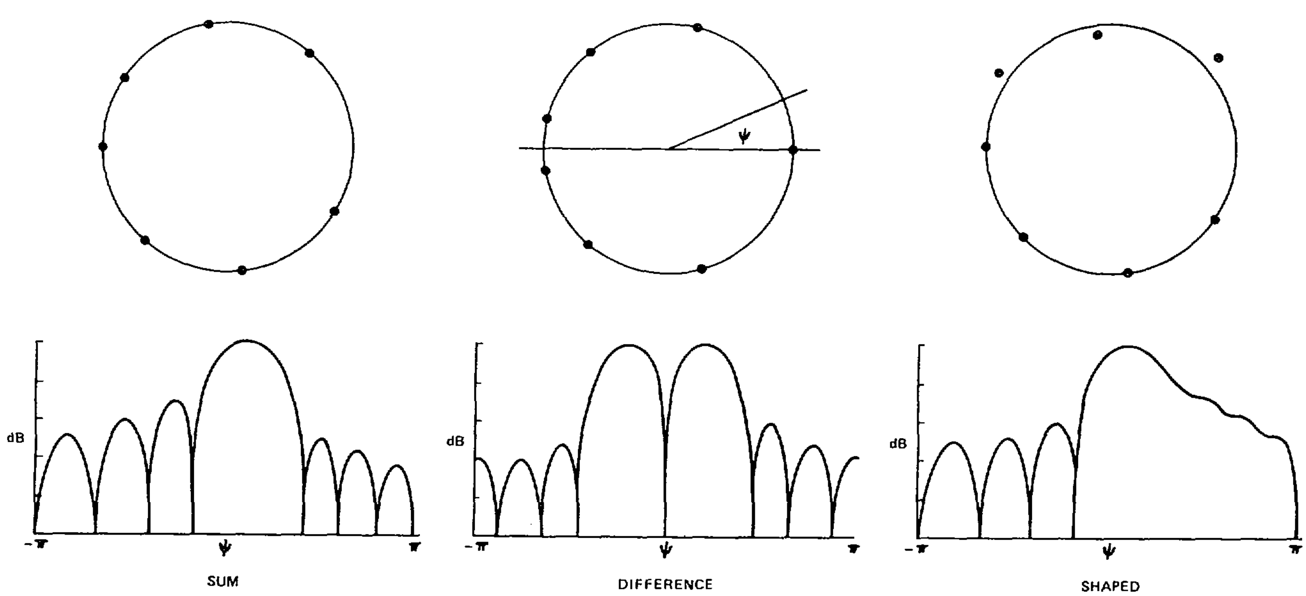


Figure 1. The Schelkunoff Unit Circle Showing Root Placement and the Corresponding Pattern for Three Cases of an Eight Element Linear Array.

any pattern producible by an equispaced linear array can be viewed as a problem of finding the proper positions of the roots w_n. Once this is done, multiplying out the factors comprising the final form of (3) will yield the penultimate form of (3), from which the relative excitations can be recognized.

Three distinct pattern types can be characterized in terms of roots on or off the Schelkunoff unit circle, as suggested in Figure 1. Several obvious features can be noted: 1) If the roots are placed on the unit circle, a pattern with deep nulls will result. 2) If successive roots are moved closer together (further apart) the intervening lobe will be lower (higher). This is the key to getting a single main beam and arbitrary side lobes in the sum pattern case, or twin main beams and arbitrary side lobes in the difference pattern case. 3) If some roots are moved off the unit circle, null-filling will result and a shaped pattern can be achieved.

From (2) we see that the total excursion of w around the unit circle, as θ goes from 0° to 180°, is $2kd = 4\pi(d/\lambda)$. If the element spacing is $d = \lambda/2$, the excursion is exactly one revolution. The antenna designer should always do pattern synthesis as though the spacing were $\lambda/2$. If the actual spacing is greater than $\lambda/2$, the total excursion of w will be greater than 2π and part of the pattern will repeat; this is unavoidable. If the actual spacing is less than $\lambda/2$, one could conceivably arrange the roots to occur in the more limited w range, but this would result in supergaining, which should be avoided at all costs.

Scanning of the pattern can be represented in either of two ways: 1) The excitations can be generalized to $I_n = I'_n \exp(-jn\alpha)$, with α a uniform progressive phase factor, and the definition of ψ can be changed to $\psi = kd\cos\theta - \alpha$. This serves the purpose of swiveling the beginning and ending positions of the w excursion, thereby shifting the pattern in θ space. 2) Equations (1)-(3) can be left intact and the set of roots w_n swivelled any desired amount. This second procedure accomplishes the same result and is computationally more convenient.

Sum Patterns -- If N+1 roots are equispaced on the unit circle, after which the root at $w = 1+j0$ is removed, we know from De Moivre's theorem that the excitation is uniform amplitude/equiphase; a sum pattern results, with symmetric side lobes that diminish in height as one departs from the main beam; the innermost pair is only down -13.5 dB. This case is important because it offers maximum directivity and can be used as a yardstick to judge other excitations.

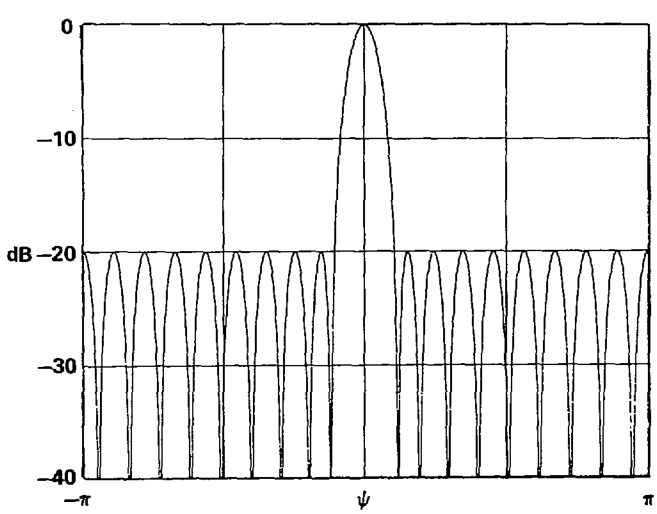
Dolph [2] was the first to find an analytic technique for controlling side lobe level. He used the properties of Chebyshev polynomials to synthesize a sum pattern with all side lobes at a common, prescribed height. His root is given by $w_n = \exp(j\psi_n)$, with ψ_n found from $u_o\cos(\psi_n/2) = \cos[(2n-1)\pi/2m]$ where $20\log_{10}T_m(u_o)$ is the side lobe level in dB and T_m is the mth Chebyshev polynomial. Simple formulas for Dolph excitations are now available in the literature [3]. A typical Dolph pattern and the array excitation which produces it are found in Figure 2. It can be observed that the excitation is tapered, with the central elements most excited, with a falling off on each side, and then with an upswing at the ends of the array. This upswing can cause mutual coupling difficulties for the antenna designer.

In the early years after Dolph's discovery, computer technology was not sufficiently advanced to produce Dolph excitations quickly and at low cost, particularly for a large number of elements. Thus array designers were gratified by the appearance of a new technique, devised by Taylor [4], which rigorously pertained only to a continuous linear aperture, but which could be adapted to arrays by sampling. Like Dolph, Taylor sought a sum pattern with symmetric side lobes at a common prescribed height. He found that this was nonrealizable with a continuous aperture, but that he could come arbitrarily close, the disparity being minor (Taylor's side lobe structure droops slightly). A Taylor sum pattern and the distribution which produces it are given by

$$S(u) = \frac{\sin\pi u}{\pi u} \cdot \frac{\prod_{n=1}^{\bar{n}-1}(1-u^2/u_n^2)}{\prod_{n=1}^{\bar{n}-1}(1-u^2/n^2)} \qquad (4)$$

$$g(\zeta) = \frac{1}{2a}\left[S(0) + 2\sum_{m=1}^{\bar{n}-1} S(m)\cos\frac{m\pi\zeta}{a}\right] \qquad (5)$$

in which $u_n = \bar{n}[A^2+(n-\frac{1}{2})^2]^{\frac{1}{2}}/[A^2+(\bar{n}-\frac{1}{2})^2]^{\frac{1}{2}}$ is a pattern root, with $\bar{n}$ a selectable integer which marks the



EXCITATION

n	I_n	n	I_n
1	.985	11	.990
2	.481	12	.959
3	.579	13	.910
4	.675	14	.844
5	.765	15	.765
6	.844	16	.675
7	.910	17	.579
8	.959	18	.481
9	.990	19	.985
10	1.000		

Figure 2. A Typical Dolph Pattern. Nineteen Elements. $d = \lambda/2$, SLL = - 20dB.

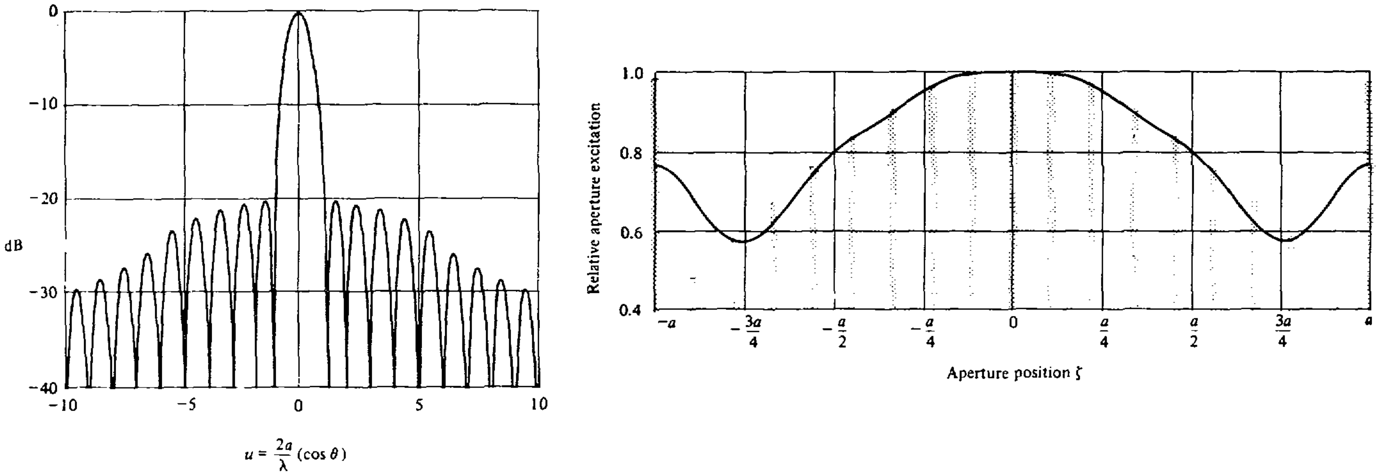


Figure 3. Typical Taylor Pattern and Corresponding Aperture Distribution. $\bar{n}$ = 6, SLL = - 20dB. Equivalent Dolph Excitation in Bar Graph Overlay.

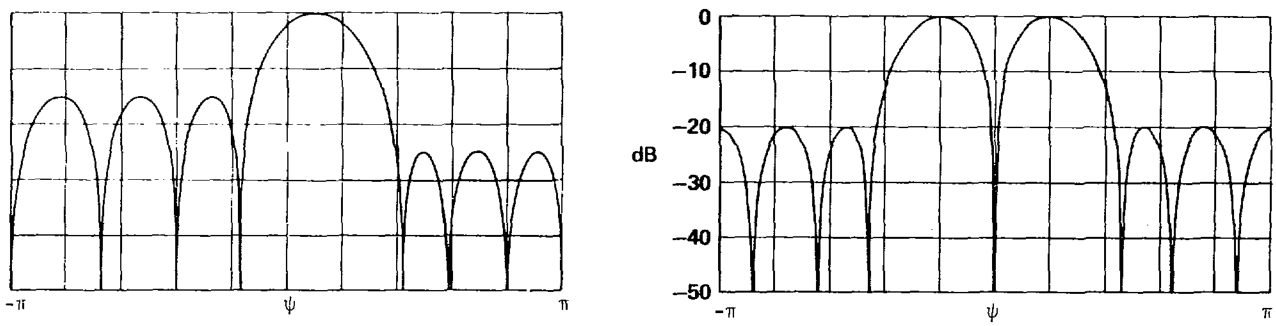


Figure 4. Array Factor for Eight Element Linear Array. (a) 15/25 dB Sum Pattern. (b) 20/20 dB Difference Pattern.

transition between the inner region where the side lobes droop slightly and the outer region where they fall off as u^{-1}. The parameter $u=(2a/\lambda)\cos\theta$ is a surrogate for the real space angular variable θ; the aperture occupies the range $-a \leq \zeta \leq a$.

A typical Taylor pattern and the corresponding aperture distribution are shown in Figure 3. As in the discrete Dolph case, we see a tapered distribution with an upswing at the ends. However, the upswing is not so great as in Dolph's case; this can be attributed to the fact that the Taylor pattern shows a drooping near-in side lobe topography. The slight loss in directivity this causes is more than offset by the easing of the mutual coupling problem.

Taylor distributions can be used in the design of equispaced linear arrays. For N+1 elements, one can sample equation (5) at N+1 equispaced points to obtain the array excitation. If N+1 is large a pattern close to (4) will result. However, for N+1 small, serious pattern degradation will occur.

Dolph and Taylor patterns suffer from the limitation that they provide uniform (or quasi-uniform) side lobe topography. Many applications do not require uniform side lobes, and since every side lobe that is held down unnecessarily is done so at the expense of main beam broadening, it is desirable to have a synthesis technique that will produce a pattern with arbitrary side lobe topography. A simple procedure [5] which is rapid and computationally inexpensive has been devised, based on linear perturbation of the root positions ψ_n. It uses the relation

$$\frac{f(w_m^p)-f_o(w_m^p)}{f_o(w_m^p)} = \frac{\delta C}{C_o} - j\sum_{n=1}^{N-1} \frac{w_n^o}{w_m^p - w_n^o}\,\delta\psi_n \qquad (6)$$

to find the shifts $\delta\psi_n$ in the root positions occasioned by going from a starting pattern $f_o(w)$ to a desired pattern $f(w)$. In (6), w_m^p is the position of the peak of the mth lobe in the starting pattern and w_n^o is the position of the nth root in the starting pattern. C_o and $C = C_o + \delta C$ are the level-adjusting factors for the starting and desired patterns so that the main lobe height of each is unity.

Matrix inversion of (6) gives the new root positions. The procedure requires iteration, but convergence is normally so rapid that one can choose as starting pattern the generic case in which the roots are equispaced. An example of a sum pattern generated in this fashion is shown in Figure 4a. Only three iterations were required to bring all side lobes within 0.1 dB of specification. The final excitation is found by multiplying out the factors shown in (3).

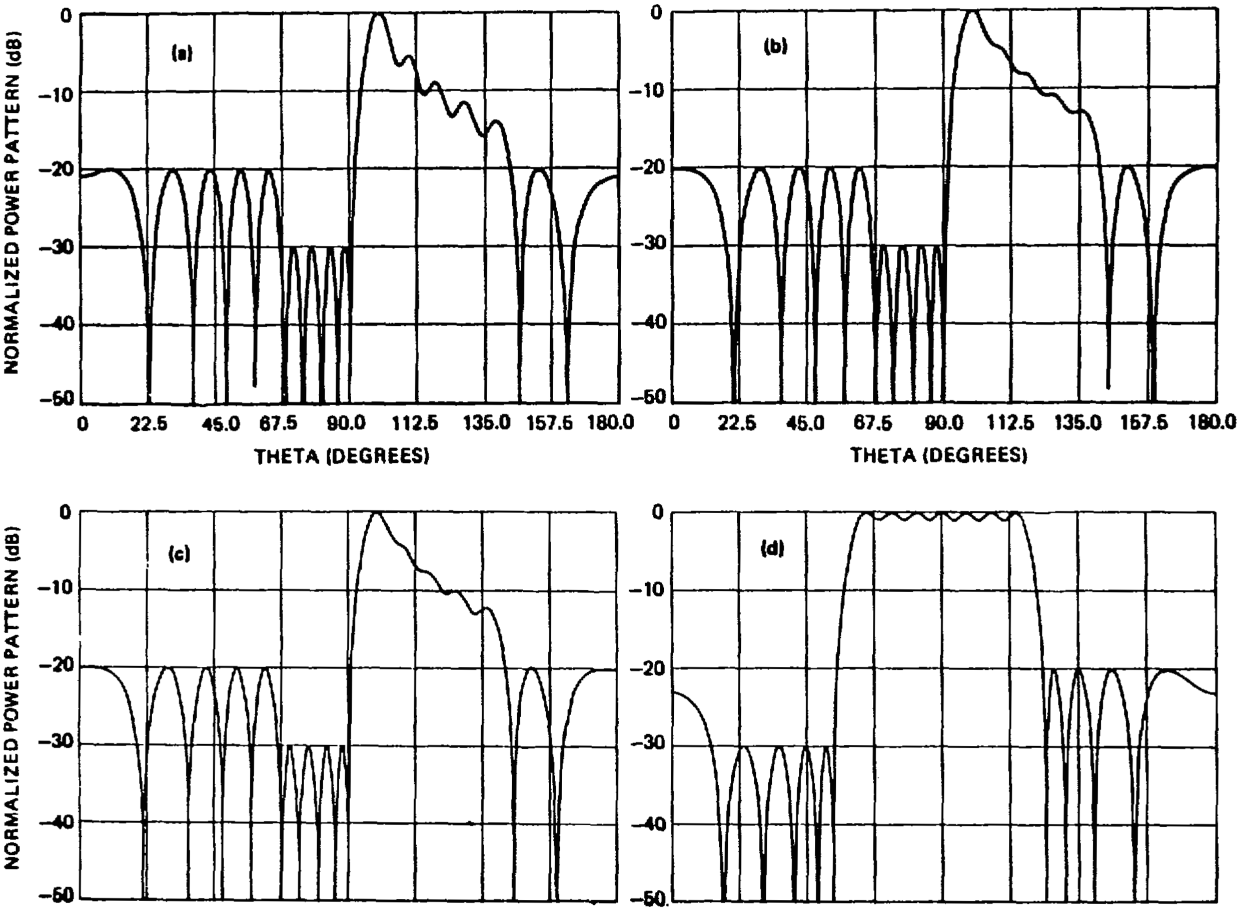


Figure 5. Four Shaped Patterns for a Sixteen Element Equispaced Linear Array

Since Dolph and Taylor patterns can be generated by this procedure (and in the Taylor case without the degradation caused by sampling), once a computer program is written to handle equation (6), the antenna designer no longer needs to utilize those earlier techniques. This is true even for N+1 large, since modern computers can invert (6) easily for N+1 values larger than one encounters in practical applications.

Difference Patterns -- Bayliss [6] developed a synthesis technique similar to Taylor's, but applicable to difference patterns. It uses a continuous linear aperture, so Bayliss distributions must be sampled in order to apply to linear arrays. Here again, pattern degradation results if N+1 is small. In recent years, little use has been made of Bayliss distributions, because (6) applies equally well to the synthesis of difference patterns. An example is shown in Figure 4b. It took but two iterations to bring all side lobes within a quarter dB of specification.

Shaped Patterns -- There are applications in which patterns devoid of nulls are desirable, such as airport beacons and ground-mapping radars. A classic approach to the synthesis of this type of pattern was developed by Woodward [7]. His original formulation was for a continuous aperture, but in its discretized form his building block is an array excitation which has uniform amplitude and uniform progressive phase. The corresponding pattern is given by [8]

$$A(\theta) = \frac{\sin[(2N+1)\pi d/\lambda(\cos\theta - \cos\theta_0)]}{\sin[\pi d/\lambda(\cos\theta - \cos\theta_0)]} \qquad (7)$$

wherein θ_0 is the angle at which the main beam points.

By superimposing a sequence of such patterns, each with its main beam coinciding with the first null of the previous pattern in the sequence, and with successive main lobe heights adjusted to fall on a desired shaping contour, Woodward was able to synthesize a pattern which approximated what was desired. Examples can be found in References 7 and 8. However, his technique has several severe limitations. First, one has no control over the ripple in the shaped region. Second, one also has no control over the side lobe level in the non-shaped region. Third, it turns out that Woodward's method makes inefficient use of the available roots, placing them in radial pairs.

Over the past forty years a number of techniques too numerous to count have been devised to improve on Woodward. By far the best of these has been formulated by Orchard [9]. He represents the starting pattern in logarithmic form, viz.,

$$G(\psi,x_i) = \sum_{n=1}^{N-1} 10\log_{10}[1-2e^{a_n}\cos(\psi-b_n) + e^{2a_n}] + 10\log_{10}[2(1+\cos\psi)] + C_1 \qquad (8)$$

with the nth root represented by $w_n = \exp(a_n+jb_n)$. The parameter x_i stands for any of the a_n or b_n. The desired pattern is represented by $S(\psi) = S_1(\psi)+C_2$. (The constants C_1 and C_2 are level-adjusting parameters.) Since the form of G is known, Orchard can construct the total differential

$$d(G-S) = \sum_i \frac{\partial G}{\partial x_i} dx_i \qquad (9)$$

The left side of (9) is known (to good approximation) at every value of ψ for which there is an extremum in G. When the partial derivatives of G which appear on the right side of (9) are determined at these values, the result is a set of linear simultaneous equations which can be inverted to give dx_i. Iteration will cause the starting pattern to converge on the desired pattern.

Several examples of Orchard's technique are given in Figure 5. Panels a, b, and c show a $\csc^2\theta\cos\theta$ shaped beam extending over a 40° sector in θ space, with four side lobes at -30 dB, the rest at -20 dB. Fig. 5a shows a ripple of ±1.5 dB, Fig. 5b a ripple of ±0.5 dB, and Fig. 5c a ripple which changes from 0.2 dB to 1.0 dB. Fig. 5d shows the design of a flat topped beam with a ripple of ±0.5 dB and 20/30 dB side lobes. In all four cases, N+1 = 16.

An interesting feature of the design of shaped beam patterns is that the roots which are off the unit circle can equally well be inside or outside. If there are M such roots, there are 2^M sets of excitations, each of which will produce the same pattern. This is of inestimable advantage when confronting mutual coupling since some of these distributions can ease that problem considerably.

Orchard's technique is computer-efficient, inexpensive, and rapidly convergent. It is so general that it can be used to synthesize any sum or difference pattern (with arbitrary side lobe topography, or Dolph, or Taylor) as well as any pattern with arbitrary shaping in one region and arbitrary side lobe topography in the remainder. Indeed, it seems fair to say that once the antenna designer has programmed equations (8) and (9), he needs no other technique to synthesize any pattern due to an equispaced linear array.

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Chapter News

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WHY CHAPTERS?

The Antennas and Propagation Society supports 31 Chapters internationally because they play a vital role in providing services to Society members at the local, Section level. Through technical and social meetings, lecture series and field trips, the Chapters provide one of the most accessible opportunities for members to gain new understandings of the emerging technologies of Society interest. The opportunity to meet and renew friendships with colleagues and discuss problems of common interest are an important aspect of the Chapter's regular meetings.

Chapters represent the Society at the local level by joining with Section officers and the officers of other Society Chapters in organizing, publicizing and hosting their technical and social meetings. These Chapter meetings, along with the Transactions, this Newsletter and the annual international Symposium are the principal Society means by which the professional standards of its members are advanced.

In recognition of the importance of Chapter activities in providing member services, the Society has several programs to support your Chapter. The Distinguished Lecturer program provides internationally known speakers on topics sure to attract interest at Chapter meetings. Professor Keith Carver has organized an outstanding program of Distinguished Lecturers for 1985-86 (they are announced elsewhere in the Newsletter) so be sure that your Chapter benefits from this program. In addition, the Society has limited funds to support some special Chapter programs such as mini-symposia, lecture series or field trips. Your Chapter Chairman