Let's apply the reciprocity theorem to model a receive antenna in terms of its properties as a transmitter.

Scenario #1 (RX)

Scenario #2 (TX)

Using the reciprocity theorem,

$$\langle \vec{J}_r, \vec{E}_t \rangle = \langle \vec{J}_t, \vec{E}_r \rangle$$

$$\langle \vec{J}_r, \vec{E}_t \rangle = \int_a^b I_r \hat{z}\, \delta(x)\delta(y)\, \vec{E}_t(x,y) = I_r v_{oc}$$

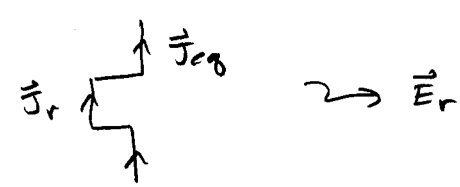
So,

$$v_{oc} = \frac{1}{I_r} \int \vec{J}_t \cdot \vec{E}_r \, d\vec{r}$$

The problem with this is that $\vec{E}_t$ includes the field scattered by the antenna. It would be simpler if this were $\vec{E}^{inc}$, the field radiated by the test source without the antenna present. Let's use the equivalence theorem to modify both scenarios:

#1 TX

#2 RX



Now, the reciprocity theorem gives

$$\langle \vec{J}_r + J_{eq}, \vec{E}^{inc} \rangle = \langle \vec{J}_t, \vec{E}^r \rangle$$

Combining the two results,

$$v_{oc} = \frac{1}{I_r} \langle \vec{J}_t, \vec{E}_r \rangle$$

$$= \frac{1}{I_r} \langle \vec{J}_r + J_{eq}, \vec{E}^{inc} \rangle$$

$$= \frac{1}{I_r} \int (\vec{J}_r + J_{eq}) \cdot \vec{E}^{inc} \, d\vec{r}$$

This provides a Thévenin equivalent for a receive antenna:

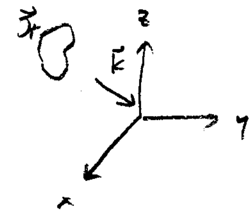
where Z_{in} is the input impedance of the antenna as a transmitter.

<u>Effective receiving area</u>

Effective area

Effective area of a receiving antenna is a very useful concept. If the transmitter is far away, then the incident field can be modeled as a plane wave:

$$\vec{E}^{inc} = \vec{E}_0 \, e^{-j\vec{k}\cdot\vec{r}},$$



$\vec{k} = -k\hat{r}$, where $\hat{r}$ points towards the transmitter

The o.c. voltage is

$$V_{oc} = \frac{1}{I_g}\int \vec{J}_{eq}(\vec{r}\,')\cdot \vec{E}_0 \, e^{-j\vec{k}\cdot\vec{r}\,'}\, d\vec{r}\,' \qquad \text{(lumping } \vec{J}_r \text{ and } \vec{J}_{eq} \text{ together)}$$

$$= \frac{1}{I_g}\vec{E}_0 \cdot \int \vec{J}_{eq}(\vec{r}\,')\, e^{-j\vec{k}\cdot\vec{r}\,'}\, d\vec{r}\,'$$

Comparing this to the radiation integral,

$$\vec{E}^r = -j\omega\mu\left(1-\hat{r}\hat{r}\cdot\right)\frac{e^{-jkr}}{4\pi r}\int \vec{J}_{eq}(\vec{r}\,')\, e^{-j\vec{k}\cdot\vec{r}\,'}\, d\vec{r}\,'$$

and forming the dot product with $\vec{E}_0$,

$$\vec{E}_0\cdot\vec{E}^r = -j\omega\mu\; \vec{E}_0\cdot\left(1-\hat{r}\hat{r}\cdot\right)\frac{e^{-jkr}}{4\pi r}\int \vec{J}_{eq}(\vec{r}\,')\, e^{-j\vec{k}\cdot\vec{r}\,'}\, d\vec{r}\,'$$

$$= -j\omega\mu\,\frac{e^{-jkr}}{4\pi r}\;\vec{E}_0\cdot\int \vec{J}_{eq}(\vec{r}\,')\, e^{-jk\hat{r}\cdot\vec{r}\,'}\, d\vec{r}\,'$$

Since $\vec{E}_0$ is orthogonal to $\hat{r}$. Combining these expressions,

$$V_{oc} = \frac{1}{I_g}\,\frac{4\pi r}{-j\omega\mu\, e^{-jkr}}\;\vec{E}_0\cdot\vec{E}^r$$

in terms of the field $\vec{E}^r$ radiated by the antenna as a transmitter.

The maximum power delivered to the load occurs when the load is conjugate matched, so that $Z_L = Z_{in}^*$, and

$$P_L = \frac{1}{2}\,\mathrm{Re}\{V_L I_L^*\} = \frac{1}{2}\,\mathrm{Re}\{Z_L |I_L|^2\}$$

$$= \frac{1}{2} R_L |I_L|^2$$

ceiv

$$= \frac{1}{2} R_{rad} \left| I_L \right|^2 \quad \text{for a conjugate match}$$

$$= \frac{1}{2} R_{rad} \left| \frac{V_{oc}}{Z_{in} + Z_{in}^*} \right|^2$$

$$= \frac{1}{2} R_{rad} \left| \frac{V_{oc}}{2 R_{rad}} \right|^2$$

$$= \frac{|V_{oc}|^2}{8 R_{rad}}$$

$$= \frac{1}{8 R_{rad}} \left| \frac{1}{I_g} \frac{4\pi r}{-j\omega\mu\, e^{-jkr}} \vec{E}_0 \cdot \vec{E}^r \right|^2$$

$$= \frac{1}{\underbrace{\frac{1}{2} R_{rad} |I_g|^2}_{P_{in}}} \frac{1}{16} \frac{(4\pi r)^2}{k^2 \eta^2} \left| \hat{E}^{inc} \cdot \hat{E}^r \right|^2 \left| \vec{E}^{inc} \right|^2 \left| \vec{E}^r \right|^2$$

(1)

$$= \frac{1}{4} \frac{1}{P_{in}} \frac{4\pi \cdot 4\pi r^2}{k^2} \left| \hat{E}^{inc} \cdot \hat{E}^r \right|^2 \underbrace{\frac{|E^{inc}|^2}{2\eta}}_{S^{inc}} \underbrace{\frac{|\vec{E}^r|^2}{2\eta}}_{S_r}$$

$$= \underbrace{\frac{\pi}{k^2}}_{\lambda^2/4\pi} \underbrace{\frac{S^r}{P_{in}/4\pi r^2}}_{G(\hat{r})\ -\ \text{gain}} \underbrace{\left| \hat{E}^{inc} \cdot \hat{E}^r \right|^2}_{\eta_{pol}} S^{inc}$$

$$= \underbrace{\frac{\lambda^2}{4\pi} G(\hat{r})\, \eta_{pol}}_{\text{Area}} \underbrace{S^{inc}}_{W/m^2}$$

This shows that the power delivered to the load is the power incident on an effective area

$$A_{eff} = \frac{\lambda^2}{4\pi} G(\hat{r})\, \eta_{pol} = \frac{\lambda^2}{4\pi} D(\hat{r})\, \eta_{tot}$$

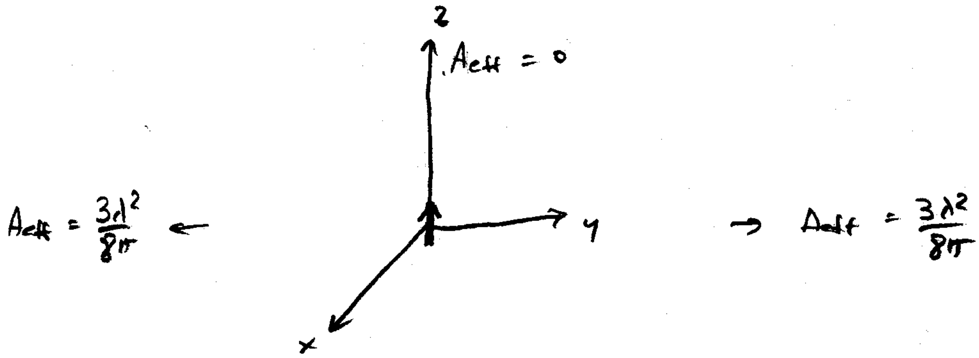
3v

This quantity is a function of the direction of the incoming wave. In a high gain direction, more power is received, so A_{eff} is bigger.

Example: Hertzian dipole

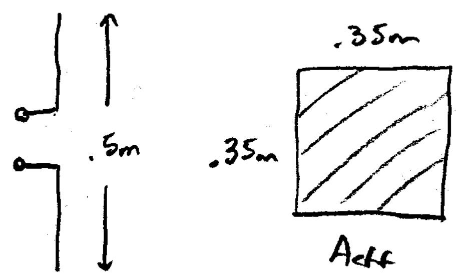
The maximum directivity of a Hertzian dipole is 1.5. This leads to an effective area of

$$A_{eff} = \frac{\lambda^2}{4\pi}\frac{3}{2} = \frac{3\lambda^2}{8\pi}$$



If $\lambda = 1m$ (300 MHz), then

$$A_{eff} = \frac{3}{8\pi} \simeq .12\ m^2 = .35m \times .35m$$



The effective area is much larger than the physical area of the antenna.