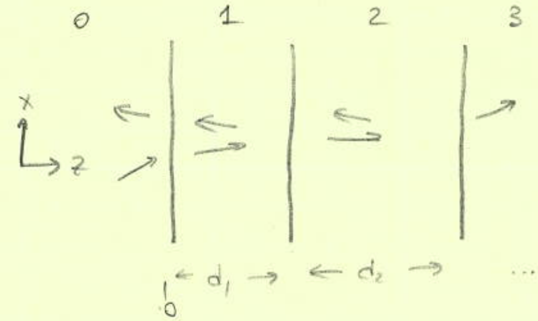
Applications: Anti-reflective coatings
Filters
Low-observable
⋮



h-pol case

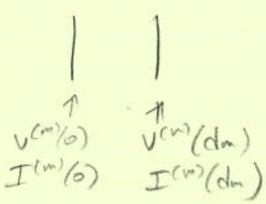
In the mth layer, the tangential field components are

$$E_y^{(m)} = \big(\underbrace{E_y^{(m)+} e^{-jk_{mz}z}}_{\text{forward wave}} + \underbrace{E_y^{(m)-} e^{jk_{mz}z}}_{\text{reverse wave}}\big) e^{-jk_x x} = V^{(m)}(z) e^{-jk_x x}$$

$$H_x^{(m)} = \frac{1}{Z^{\perp(m)}} \left(E_y^{(m)+} e^{-jk_{mz}z} - E_y^{(m)-} e^{jk_{mz}z} \right) e^{-jk_x x} = I^{(m)}(z) e^{-jk_x x}$$

$$Z^{\perp(m)} = \frac{Z_F^{(m)}}{\cos\theta_m}$$

These expressions have the form of solutions to the 1D transmission line differential equations.



$$E_y^{(m)+}(0)\, e^{-jk_{mz}d_m} + E_y^{(m)-} e^{jk_{mz}d_m} = V^{(m)}(d_m)$$

$$E_y^{(m)+}(0)\, e^{-jk_{mz}d_m} - E_y^{(m)-} e^{jk_{mz}d_m} = Z^{\perp(m)} I^{(m)}(d_m)$$

$$E_y^{(m)+}(0) + E_y^{(m)-}(0) = V^{(m)}(0)$$

$$E_y^{(m)+}(0) - E_y^{(m)-}(0) = Z^{\perp(m)} I^{(m)}(0)$$

$[m] = \begin{bmatrix} e & \cdots \\ e & \cdots \end{bmatrix} \begin{matrix} E \\ E \end{matrix}$

$[0] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{matrix} E \\ E \end{matrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} e & e \\ e & e \end{bmatrix}^{-1} [\ \ $

$$\begin{bmatrix} V^{(m)}(0) \\ Z^{\perp(m)} I^{(m)}(0) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} e^{-j\phi} & e^{j\phi} \\ e^{-j\phi} & -e^{j\phi} \end{bmatrix}^{-1} \begin{bmatrix} V^{(m)}(d_m) \\ Z^{\perp(m)} I^{(m)}(d_m) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{\begin{bmatrix} -e^{j\phi} & -e^{j\phi} \\ -e^{-j\phi} & e^{-j\phi} \end{bmatrix}}{-1 \ -1} \begin{bmatrix} V^{(m)}(d_m) \\ Z^{\perp(m)} I^{(m)}(d_m) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & +j\sin\phi \\ +j\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \ \ " \ \ \end{bmatrix}$$

So,

$$\begin{bmatrix} V^{(m)}(0) \\ I^{(m)}(0) \end{bmatrix} = \underbrace{\begin{bmatrix} \cos k_{mz} d_m & j Z^{\perp(m)} \sin k_{mz} d_m \\ \frac{j}{Z^{\perp(m)}} \sin k_{mz} d_m & \cos k_{mz} d_m \end{bmatrix}}_{A^{\perp(m)}} \begin{bmatrix} V^{(m)}(d_m) \\ I^{(m)}(d_m) \end{bmatrix}$$

We can solve the structure using

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \underbrace{A^{\perp(1)} A^{\perp(2)} \cdots A^{\perp(n)}}_{A^{\perp}} \begin{bmatrix} V(d) \\ I(d) \end{bmatrix}$$

In region $n+1$, there is no forward wave, so $V(d) = Z^{\perp(n+1)} I(d)$, and

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} A_{11}^{\perp} & A_{12}^{\perp} \\ A_{21}^{\perp} & A_{22}^{\perp} \end{bmatrix} \begin{bmatrix} Z^{\perp(n+1)} I(d) \\ I(d) \end{bmatrix}$$

$$Z^{\perp}(0) = \frac{V(0)}{I(0)} = \frac{A_{11}^{\perp} Z^{\perp(n+1)} + A_{12}^{\perp}}{A_{21}^{\perp} Z^{\perp(n+1)} + A_{22}^{\perp}}$$

This is the input impedance of the structure in the t-line analogy. So,

$$R^{\perp} = \frac{Z^{\perp}(0) - Z^{\perp(0)}}{Z^{\perp}(0) + Z^{\perp(0)}}$$

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} 1 + R^{\perp} \\ \frac{1 - R^{\perp}}{Z^{\perp(0)}} \end{bmatrix} = A^{\perp} \begin{bmatrix} V(d) \\ I(d) \end{bmatrix}$$

$$\begin{bmatrix} T^{\perp} \\ T^{\perp}/Z^{\perp(n+1)} \end{bmatrix} = A^{\perp -1} \begin{bmatrix} 1 + R^{\perp} \\ \frac{1 - R^{\perp}}{Z^{\perp(0)}} \end{bmatrix}$$

V-pol

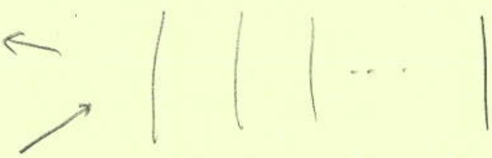
$$E^{\parallel(m)} = V^{\parallel(m)}(z)\, e^{-jk_x x}\, d_x + (\quad)\, d_z$$

$$H^{\parallel(m)} = I^{\parallel(m)}(z)\, e^{-jk_x x}\, d_y$$

$$V^{\parallel(m)}(z) = E_{x_0}^{(m+)} e^{-jk_{mz}z} + E_{x_0}^{(m-)} e^{jk_{mz}z}$$

$$I^{\parallel(m)}(z) = \frac{1}{Z^{\parallel(m)}}\left(E_{x_0}^{(m+)} e^{-jk_{mz}z} - E_{x_0}^{(m-)} e^{jk_{mz}z}\right)$$

$$\begin{bmatrix} V^{\parallel}(0) \\ I^{\parallel}(0) \end{bmatrix} = A^{\parallel(1)} \cdots A^{\parallel(n)} \begin{bmatrix} V^{\parallel}(d) \\ I^{\parallel}(d) \end{bmatrix}$$



$$\vec{E}^{\,i}(0) = (\underbrace{E_{x_0}^{(0+)}}_{1} + \underbrace{E_{x_0}^{(0-)}}_{R})\, e^{-jk_x x}\, d_x + (\cdot)\, d_z$$

$$V^{\parallel}(0) = 1 + R$$

$$H^{i}(0) = \frac{1}{Z^{\parallel(0)}}(\underbrace{E_{x_0}^{(0+)}}_{1} - \underbrace{E_{x_0}^{(0-)}}_{R})\, e^{-jk_x x}\, d_y$$

$$E(d) = (\underbrace{E_{x_0}^{(n+1+)} e^{-jk_{n+1\,z}d}}_{T} + E_{x_0}^{(n+1-)} e^{jk_{mz}d})\, e^{-jk_x x} + (\;)\, d_z \quad \left(E_{x_0}^{(n+1-)} \to 0\right)$$

$$H(d) = \frac{1}{Z^{\parallel(n+1)}}(\;T \qquad - \qquad)$$

$$V^{\parallel}(d) = T$$

$$I^{\parallel}(d) = T/Z^{\parallel(n+1)}$$

$$\begin{bmatrix} 1+R^{\parallel} \\ \frac{1-R^{\parallel}}{Z^{\parallel(0)}} \end{bmatrix} = A \begin{bmatrix} T \\ T/Z^{\parallel(n+1)} \end{bmatrix}$$