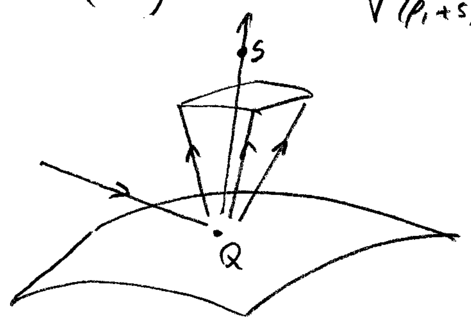
Geometrical optics (GO) is a high frequency approximation for scattering from a curved surface. The form for the scattered field is

$$\vec{E}^r(s) = \vec{E}^i(Q) \cdot \bar{\bar{R}} \sqrt{\frac{\rho_1 \rho_2}{(\rho_1+s)(\rho_2+s)}} \; e^{-j\beta s}$$



ρ_1, ρ_2 = radii of curvature of the reflected wave front

$\bar{\bar{R}}$ = dyad of Fresnel reflection coefficients

<u>Example:</u> Sphere

In the far field,

$$\sqrt{\frac{\rho_1 \rho_2}{(\rho_1+s)(\rho_2+s)}} \rightarrow \frac{\sqrt{\rho_1 \rho_2}}{s}$$

If the incident field is a plane wave, then

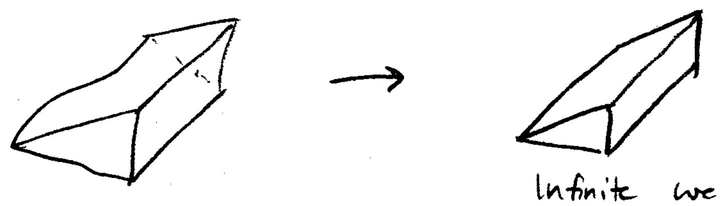
$$\rho_1 \rho_2 = \frac{R_1 R_2}{4}$$

where R_1 and R_2 are the radii of curvature of the surface at Q. So,

$$E^r = E_0 \, (-1) \sqrt{\frac{a^2}{4}} \; e^{-j\beta s}$$

$$\sigma = \lim_{s \to \infty} 4\pi s^2 \frac{\left| -E_0 \frac{a}{2} e^{-j\beta s} \right|^2}{|E_0|^2} = \pi a^2 \quad = \text{cross sectional area}$$

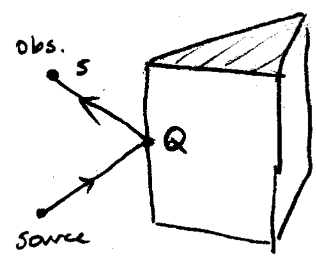
The basic idea of GTD is to add edge diffraction to geometrical optics. This is done by modeling edges as a canonical problem with asymptotic solution:



Infinite wedge

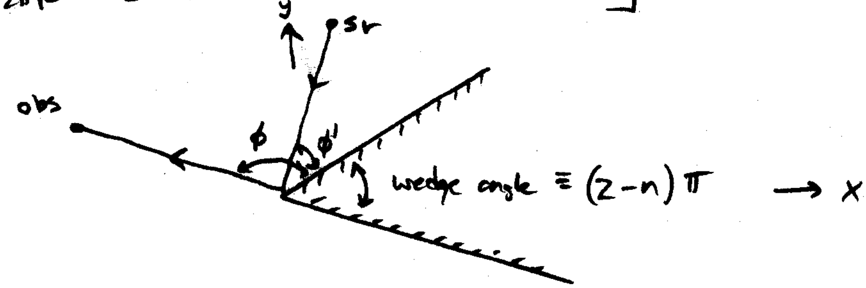
The diffracted field from a wedge is

$$\vec{E}^d(s) = \underbrace{\vec{E}^i(Q)}_{\text{field at vertex}} \cdot \underbrace{\bar{D}}_{\text{diffraction coefficient}} \; \underbrace{A(s',s)}_{\text{spreading factor} \,=\, \frac{1}{\sqrt{s}} \text{ for plane wave incident field}} e^{iks}$$



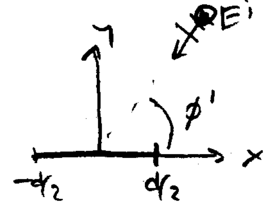
For the TM polarization, $\vec{E}$ is in the z direction, and $\bar{D}$ reduces to

$$D = \frac{\sqrt{i}\,\frac{\sin(\pi/n)}{n}}{\sqrt{2\pi k}} \left[\frac{1}{\cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\phi-\phi'}{n}\right)} + \frac{1}{\cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\phi+\phi'}{n}\right)} \right]$$



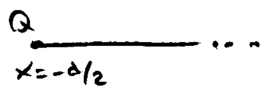
Example Flat strip of width d

$\vec{E}^i = \hat{z}\, e^{+i\vec{k}^i\cdot\vec{r}}$, $\vec{k}^i = -k\cos\phi'\hat{x} - k\sin\phi'\hat{y}$



We will find the scattering width (2D RCS) in the GTD approximation.

The strip has two edges, so we need two wedges. Let's start with the one at $x = -d/2$:



wedge angle $= 0 = (2-n)\pi \Rightarrow n = 0$

$$E^d_{1z} = E^i_z(Q_1)\cdot D_1\, \frac{e^{ik\rho_1}}{\sqrt{\rho_1}}$$

where

$$D_1 = \frac{\sqrt{i}}{\sqrt{2\pi k}\,2}\left[\frac{1}{-\cos\left(\frac{\phi-\phi'}{2}\right)} + \frac{1}{-\cos\left(\frac{\phi+\phi'}{2}\right)}\right]$$

For backscattering, $\phi = \phi'$, so

$$D_1 = -\sqrt{\frac{i}{2\pi k}}\,\frac{1}{2}\left[1 - \frac{1}{\cos(\phi)}\right]$$

The incident field is $E^i_z(Q_1) = e^{i\vec{k}^i\cdot\vec{r}}\Big|_{x=-d/2} = e^{ik\cos\phi\, d/2}$

and $\rho_1 = |\vec{r} - (-\hat{x}d/2)| \simeq \rho + \frac{d}{2}\cos\phi$.

Similarly, for the second wedge

$\phi \rightarrow \pi - \phi$ (since flipped...)

$$D_2 = -\sqrt{\frac{i}{2\pi k}}\,\frac{1}{2}\left[1 + \frac{1}{\cos\phi}\right]$$

$$\rho_2 = \rho - \frac{k}{2}\cos\phi$$

The two wedge contributions are

$$E^d_{1z} = \left[e^{ik\cos\phi\, d/2}\right]\left[-\sqrt{\frac{i}{2\pi k}}\,\frac{1}{2}\left(1 - \frac{1}{\cos\phi}\right)\right]\frac{e^{ik(\rho + \frac{d}{2}\cos\phi)}}{\sqrt{\rho}}$$

$$E^d_{2z} = \left[e^{-ik\cos\phi\, d/2}\right]\left[\quad '' \quad \left(1 + \frac{1}{\cos\phi}\right)\right]\frac{e^{ik(\rho - \frac{d}{2}\cos\phi)}}{\sqrt{\rho}}$$

Simplifying,

$$E_{1z}^{d} = -\frac{1}{2}\sqrt{\frac{i}{2\pi k}}\left(1-\frac{1}{\cos\phi}\right)\frac{e^{ik\rho}}{\sqrt{\rho}}\,e^{ikd\cos\phi}$$

$$E_{2z}^{d} = -\frac{1}{2}\sqrt{\frac{i}{2\pi k}}\left(1+\frac{1}{\cos\phi}\right)\frac{e^{ik\rho}}{\sqrt{\rho}}\,e^{-ikd\cos\phi}$$

Simplifying again,

$$E_{1z}^{d} = -\sqrt{\frac{2i}{\pi k\rho}}\,e^{ik\rho}\,\frac{i}{4}\left(1-\frac{1}{\cos\phi}\right)e^{ikd\cos\phi}$$

$$E_{2z}^{d} = -\sqrt{\frac{2i}{\pi k\rho}}\,e^{ik\rho}\,\frac{i}{4}\left(1+\frac{1}{\cos\phi}\right)e^{-ikd\cos\phi}$$

Adding the two,

$$\vec{E}^{d} = -\sqrt{\frac{2i}{\pi k\rho}}\,e^{ik\rho}\,\frac{i}{4}\left[\left(1-\frac{1}{\cos\phi}\right)e^{ik_x d} + \left(1+\frac{1}{\cos\phi}\right)e^{-ik_x d}\right], \quad k_x = k\cos\phi$$

$$= \sqrt{\frac{2i}{\pi k\rho}}\,e^{ik\rho}\left[-\frac{i}{4}\left\{2\cos(k_x d) - 2i\,\frac{\sin(k_x d)}{\cos\phi}\right\}\right]$$

$$= \quad '' \quad \underbrace{\left[-i\,\frac{\cos(k_x d)}{2} - \frac{\sin(k_x d)}{2\cos\phi}\right]}_{\text{monostatic scattering amplitude}}$$ *

How does this relate to the PO approximation?