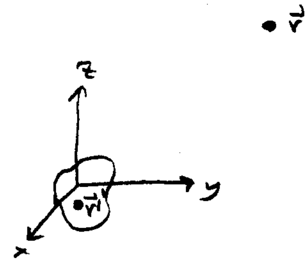
If the observation point (or field point) is far away from the source, then we can simplify the radiation integral

$$\vec{E}(\vec{r}) = i\omega\mu\left[\bar{\bar{I}} + \frac{1}{k^2}\nabla\nabla\right]\cdot\int d\vec{r}'\, \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\vec{J}(\vec{r}') .$$



If $r \gg r'$, then

$$|\vec{r}-\vec{r}'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\simeq \sqrt{x^2 - 2xx' + y^2 - 2yy' + z^2 - 2zz'}$$

$$= \sqrt{r^2 - 2(xx' + yy' + zz')}$$

$$= r\sqrt{1 - \frac{2}{r^2}(xx' + yy' + zz')}$$

$$\simeq r\left(1 - \frac{1}{r^2}(xx' + yy' + zz')\right)$$

$$= r - \frac{1}{r}(\vec{r}\cdot\vec{r}')$$

$$= r - \underline{\underline{\hat{r}\cdot\vec{r}'}} \qquad \text{since} \quad \hat{r} = \vec{r}/r .$$

This is the 'far field' approximation. Now, consider

$$g(\vec{r},\vec{r}') = \frac{e^{ik|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}$$

$$\simeq \frac{e^{ik(r-\hat{r}\cdot\vec{r}')}}{4\pi(r-\hat{r}\cdot\vec{r}')}$$

$$= \frac{e^{ikr}e^{-ik\hat{r}\cdot\vec{r}'}}{4\pi r\,(1-\hat{r}\cdot\vec{r}'/r)}$$

$$\simeq \frac{e^{ikr}}{4\pi r}e^{-ik\hat{r}\cdot\vec{r}'}\,(1 + \underbrace{\hat{r}\cdot\vec{r}'/r}_{\text{second order term — must drop it...}})$$

$$\cong \frac{e^{ikr}}{4\pi r}e^{-ik\hat{r}\cdot\vec{r}'}$$

Note that this amounts to using the approximations

$$1^{st}\text{ order:}\quad |\vec{r}-\vec{r}\,'| \simeq r - \hat{r}\cdot\vec{r}\,' \qquad \text{in the phase}$$

$$0^{th}\text{ order:}\quad |\vec{r}-\vec{r}\,'| \simeq r \qquad \text{in the denominator}$$

Physically, small changes in the denominator lead to small changes in the value of the function, whereas small changes in the phase are significant.

Using this result in the radiation integral,

$$\vec{E}(\vec{r}) \simeq i\omega\mu\left[\overline{\overline{I}} + \frac{1}{k^2}\nabla\nabla\right]\cdot\frac{e^{ikr}}{r}\int d\vec{r}\,'\,\vec{J}(\vec{r}\,')\,e^{-ik\hat{r}\cdot\vec{r}\,'}$$

The $\nabla\nabla$ term becomes

$$\nabla\nabla\frac{e^{ikr}}{r} = \nabla\left(\hat{r}\frac{\partial}{\partial r}\frac{e^{ikr}}{r} + O(1/r^2)\right)$$

$$= \hat{r}\frac{\partial^2}{\partial r^2}\hat{r}\frac{e^{ikr}}{r} + O(1/r^2)$$

$$= (ik)^2\,\hat{r}\hat{r}\,\frac{e^{ikr}}{r}$$

$$= -k^2\frac{e^{ikr}}{r}\,\hat{r}\hat{r}$$

The radiation integral becomes

$$\vec{E}(\vec{r}) \simeq i\omega\mu\left[\overline{\overline{I}} - \hat{r}\hat{r}\right]\frac{e^{ikr}}{r}\int d\vec{r}\,'\,\vec{J}(\vec{r}\,')\,e^{-ik\hat{r}\cdot\vec{r}\,'}$$

The $\hat{r}\hat{r}$ term 'subtracts' the longitudinal wave, or the wave in the $\hat{r}$ direction at the observation point.
The integral

$$\int d\vec{r}\,'\,\vec{J}(\vec{r}\,')\,e^{-ik\hat{r}\cdot\vec{r}\,'}$$

is referred to as the vector current moment. This can be computed first, and then used to find $\vec{E}$ in the far field.