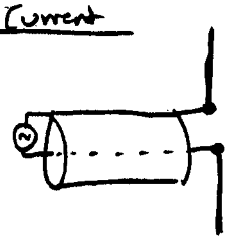
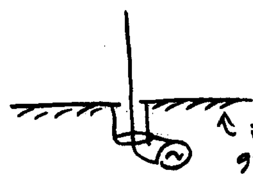
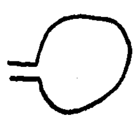
An antenna transforms energy from a transmission line or a waveguide to a wave propagating in free space.

Examples:

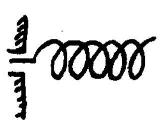
Current

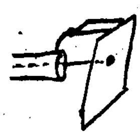
Dipole: 

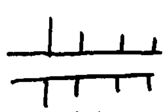
Monopole:  PEC ground plane

Loop: 

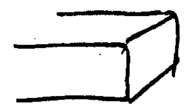
Patch:  PCB

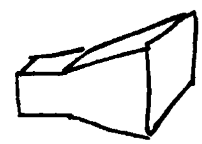
Helical:  (circular pol.)

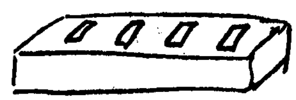
"Planar inverted F": (cell phone) 

Yagi:  (directional)

Aperture

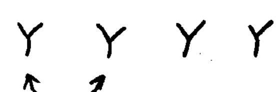
Open-ended waveguide: 

Horn:  (narrower pattern)

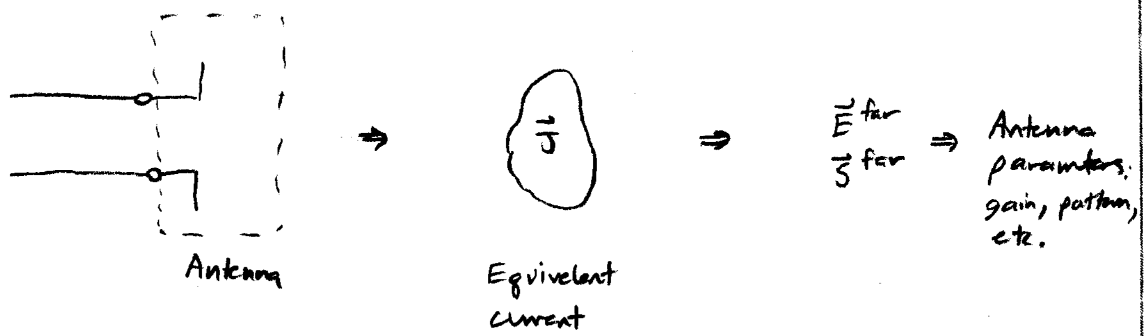
Slotted waveguide: 

Other types: Reflector: 

(very narrow pattern)

Array: 
Any of the above antennas
(shapeable beam, steerable..)

Goal of antenna analysis:



1. Find an equivalent current.

   Approaches: Approximations
   - Linear antennas: Hertzian dipole model, triangular current model, sinusoidal, ...
   - Aperture antennas: aperture field $\simeq$ incident field
   - etc.

   Numerical method (ECEn 563)

2. Find far fields using radiation integral

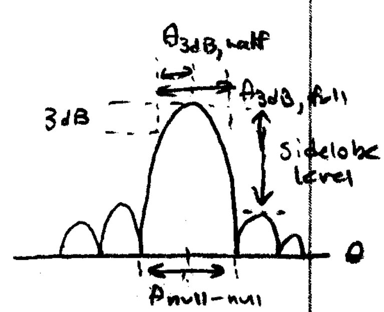
3. Find antenna parameters from $\vec{S}^{\,far}$

Antenna design = inverse problem: given antenna parameters, design the antenna. (More difficult).

Basic antenna parameters are:

Radiation pattern, gain, directivity
main lobe beamwidth, sidelobe levels

Radiation resistance
Efficiency
Bandwidth, frequency of operation, size



Radiation pattern

$$\vec{S} = E \times H^* \rightarrow \underline{\underline{f(\theta,\phi)}}\,\frac{1}{r^2}\,\hat{r}, \quad r \rightarrow \infty$$

$\hookrightarrow$ radiation pattern $= \dfrac{f(\theta,\phi)}{f_{max}}$

Gain/directivity

Power radiated to far field: $P_{rad} = \oint_S \vec{S}_{av} \cdot d\vec{s}$

Directivity: $D(\theta,\phi) = \dfrac{\text{Power density at } (\theta,\phi)}{\text{Power density of isotropic radiator}}$

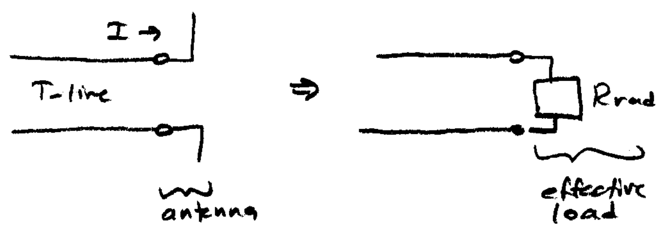
$$= \frac{S^{ff}_{av,r}}{P_{rad}/4\pi r^2}, \quad S^{ff}_{av,r} = \frac{1}{2}\,\mathrm{Re}\left[E \times H^*\right] \cdot \hat{r}, \text{ large } r \ldots \text{ (far field)}$$

Efficiency: $\eta_{rad} = \dfrac{P_{rad}}{P_{in}}$, $\eta_{rad} < 1$ if antenna is lossy

Gain: $G(\theta,\phi) = \eta_{rad}\, D(\theta,\phi)$

Radiation resistance

$$R_{rad} = \frac{P_{rad}}{\frac{1}{2}|I|^2}$$



Bandwidth/frequency/size

Resonant antenna: desirable radiation resistance over some band

Low frequencies: $R_{rad} \sim (kR)^2$, so antenna must be large or else efficiency is low

Antenna $\ll \lambda$: broad pattern, low gain

Antenna $\gg \lambda$: narrow beam, high gain - 3 m dish at 1.5 GHz $\cong$ 30 dB gain

Example: Hertzian dipole

For the Hertzian dipole,

$$\vec{S} = \frac{1}{2}\mathrm{Re}\, \vec{E} \times \vec{H}^*$$

$$= -i\omega\mu \frac{e^{ikr}}{4\pi r} \hat{\theta}\, I\ell \sin\theta \times \left(-\frac{i\omega\mu}{\eta} \frac{e^{+ikr}}{4\pi r} I\ell \sin\theta\, \hat{\phi} \right)^*$$

$$= \frac{(\omega\mu)^2}{\eta} \frac{(I\ell)^2}{(4\pi r)^2} \sin^2\theta\, \hat{r}$$

$$= \frac{\eta}{2} \left(\frac{kI\ell}{4\pi r} \right)^2 \sin^2\theta\, \hat{r}$$

The total power radiated is

$$P_{rad} = \frac{1}{2}\int_0^{2\pi} \int_0^{\pi} r^2 \sin\theta\, d\theta\, d\phi\; \eta \left(\frac{kI\ell}{4\pi r} \right)^2 \sin^2\theta$$

$$= \eta \left(\frac{kI\ell}{4\pi} \right)^2 2\pi \int_0^{\pi} \sin^3\theta\, d\theta$$

$$= \eta \left(\frac{kI\ell}{4\pi} \right)^2 \cdot \frac{8\pi}{3}$$

$$= \eta \frac{(kI\ell)^2}{12\cancel{8}\pi}$$

The gain pattern is

$$G(\theta,\phi) = \frac{S(\theta,\phi)}{P_{rad}/4\pi r^2}$$

$$= \frac{4\pi r^2 \left(\eta \left(\frac{kI\ell}{4\pi r} \right)^2 \sin^2\theta \right)}{\eta \left(\frac{kI\ell}{4\pi} \right)^2 \cdot \frac{8\pi}{3}}$$

$$= \left(\frac{4\pi}{\frac{8\pi}{3}} \right) \cdot \sin^2\theta$$

$$= \frac{3}{2} \sin^2\theta$$

where we assume no loss, $\eta_r = 1$.

The radiation resistance is

$$R_r = \frac{P_{rad}}{\frac{1}{2} I_o^2}$$

$$= \frac{\eta \frac{(kI_o\ell)^2}{6\pi \cdot 2}}{\frac{1}{2} I_o^2}$$

$$= \eta \frac{(k\ell)^2}{6\pi}$$

$$\simeq 120\pi \frac{(k\ell)^2}{6\pi}$$

$$= \underline{20(k\ell)^2}$$

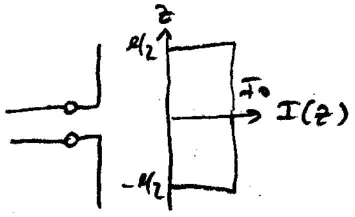
This result holds only for small $k\ell$. The reason for this is that if we drive a long wire antenna, the current is not uniform along the wire, so the Hertzian dipole model breaks down.

Hertzian dipole

The Hertzian dipole is

$$\uparrow z \qquad \begin{matrix} \oplus\ q e^{i\omega t} \\ \updownarrow \ell \\ \ominus -q e^{i\omega t} \end{matrix} \quad \Rightarrow \quad \vec{J} = \delta(x)\,\delta(y)\,\delta(z)\,\hat{z}\, I\ell$$

As a linear or wire antenna, this corresponds to the model



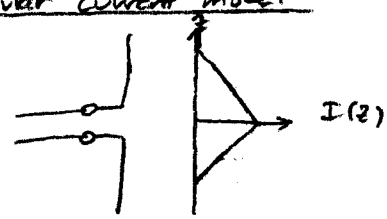
(ℓ = full length of antenna)

where the wires are short ($I \ll \lambda$) and the current on the wires is constant. This leads to

$$R_{rad} \cong 20\,(k\ell)^2 \quad \cong 200\,\Omega \text{ for half-wave dipole, } \ell = \lambda/2$$

But in reality the current $I(z)$ must vanish at the ends of the wire, so a better current model would be more accurate.

Triangular current model



$$\vec{J} = \hat{z}\,\delta(x)\,\delta(y)\,I_0\left(1 - |z|/(\ell/2)\right), \quad |z| < \ell/2$$

For this current model,

$$\bar{f}_i = \int d\vec{r}'\, \underbrace{e^{-i\vec{k}\cdot\vec{r}}}_{\to 1 \text{ for } \ell \ll \lambda}\, \vec{J}(\vec{r})\,\hat{z}$$

$$= \int_{-\ell/2}^{\ell/2} dz\, \left(1 - |z|/(\ell/2)\right) I_0\,\hat{z}$$

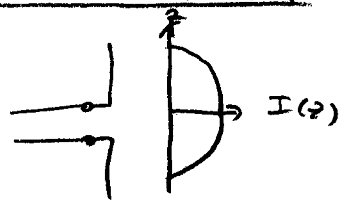
$$= \frac{I_0 \hat{z}}{2} \quad = \frac{1}{2}\left(\bar{f}_{HD}\right)$$

Because of the 1/2 factor, R_{rad} decreases by 1/4:

$$R_{rad} \cong 5\,(k\ell)^2 \qquad \text{(triangle current model)}$$

$$\cong 50\,\Omega \text{ for half-wave dipole}$$

Sinusoidal current model

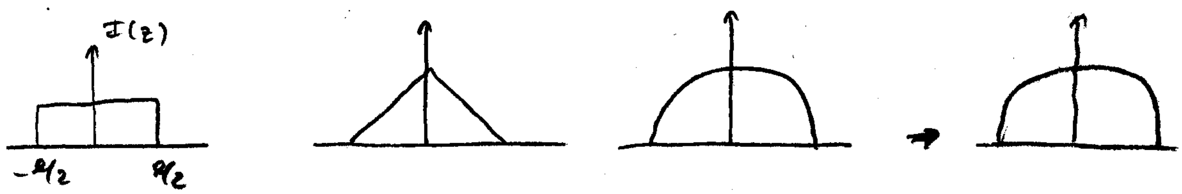


$$\vec{J} = \hat{z}\,\delta(x)\,\delta(y)\,I_0 \sin\left[k\left(\ell/2 - |z|\right)\right]$$

Evaluating $\vec{S}^{far}$ and finding the radiation resistance is

$$R_{rad} \approx 80\,\Omega \text{ for half-wave dipole}$$

What is the "true" value of R_{rad}?



Hertzian dipole	Triangular model	Sinusoidal	true current
$200\,\Omega$	$50\,\Omega$	$80\,\Omega$	
		(w/ phase, $73\,\Omega$)	

Other improvements to the model include finite wire thickness, balun, etc...