ECEn 560
Electromagnetic Wave Theory

Homework #7
Due Jan. 28, 2016 (may be turned in late for half credit)

1. (a) Represent an arbitrary 3×3 matrix A as a dyad $\overline{\overline{A}}$ (i.e., a linear combination of products $\hat{x}\hat{x}$, $\hat{x}\hat{y}$, etc). (b) Verify that $\overline{\overline{A}} \cdot \overline{b}$ gives the same result as matrix vector multiplication Ab, where b is a column vector. (c) Is the dyad $\hat{x}\hat{y}$ equal to the dyad $\hat{y}\hat{x}$?

2. Write down the physical meaning of the various terms in the far field radiation integral

$$\overline{E}(\overline{r}) = i\omega\mu(1 - \hat{r}\hat{r}\cdot)\frac{e^{ikr}}{4\pi r}\int e^{jk\hat{r}\cdot\overline{r}'}\overline{J}(\overline{r}')\,d\overline{r}'$$

3. Find the electric far field radiated by the current source $\overline{J}(\overline{r}) = \hat{z}\delta(\overline{r})$.

4. Find the error term for the approximation in the phase used in deriving the far field radiation integral. Under what conditions on r and $\overline{r}'$ is this term much smaller than 2π? From this, find a condition on the maximum diameter of a source and the distance of the observation point from the source for which the far field is valid.

5. A convenient form of the far field radiation integral is

$$\overline{E}(\overline{r}) = i\omega\mu\frac{e^{ikr}}{4\pi r}(\hat{\theta}f_\theta + \hat{\phi}f_\phi)$$

where $\overline{f}$ is the vector current moment. (a) Derive this equation from the radiation integral. (b) Derive a similar expression for the far field of a magnetic current source.