

**ECEn 560**  
**Electromagnetic Wave Theory**

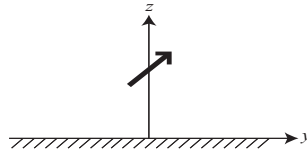
Homework #15

Due March 8, 2016 (may be turned in late for half credit)

1. For each part, give the answer to the question and also state the name of the theorem used to solve the problem.

a. An electric current  $\bar{J}_0$  A/m radiates a field  $\bar{E} = \hat{\theta}\mu e^{-j\beta r}/r$ . Find the magnetic field radiated by a magnetic current  $\bar{M} = \bar{J}_0$  Wb/m.

b. A Hertzian dipole sits a distance  $d$  above an infinite PEC sheet. The plane is on the  $z = 0$  plane, and the dipole is directed  $45^\circ$  from the  $z$  axis. Find a source which radiates the same fields for  $z > 0$  in free space.



c. A source  $\bar{J}$  sits inside the unit sphere. The fields radiated by the source in free space are  $\bar{E}_1$  and  $\bar{H}_1$ .

i. Find surface currents  $\bar{J}_s$  and  $\bar{M}_s$  on the surface of the unit sphere in terms of  $\bar{E}_1$  and  $\bar{H}_1$  which radiate the same fields for  $r > 1$  and zero fields for  $r < 1$ .

ii. Find surface currents  $\bar{J}_s$  and  $\bar{M}_s$  on the surface of the unit sphere which radiate the same fields for  $r > 1$  if the unit sphere is filled with PEC.

iii. Find surface currents  $\bar{J}_s$  and  $\bar{M}_s$  on the surface of the unit sphere which radiate the same fields for  $r > 1$  if the unit sphere is filled with PMC.

d. A source  $\bar{J}$  radiates fields  $\bar{E}_1$  and  $\bar{H}_1$  in free space. A PEC sphere is then placed at the origin, and the radiated fields change to  $\bar{E}$  and  $\bar{H}$ . Find surface currents  $\bar{J}_s$  and  $\bar{M}_s$  which when impressed on the surface of the PEC sphere radiate the fields  $\bar{E}^s = \bar{E} - \bar{E}_1$  and  $\bar{H}^s = \bar{H} - \bar{H}_1$  outside the sphere.

e. For the setup of (d), find the surface currents which radiate the fields  $\bar{E}^s$  and  $\bar{H}^s$  in free space.

2. (a) Find an electric surface current  $\bar{J}_s$  on the  $z = 0$  plane that radiates a plane wave traveling at an angle  $\theta$  with respect to the  $z$  axis for  $z > 0$ . (b) What is the field radiated by this current for  $z < 0$ ? (c) Find magnetic and electric surface currents that radiate the same plane wave for  $z > 0$  and zero fields for  $z < 0$ .

3. Find an equivalent current source in free space that radiates the same field as an open ended rectangular waveguide antenna excited in the dominant mode.