ECEn 560
Electromagnetic Wave Theory

Homework #13
Due Mar. 1, 2016 (may be turned in late for half credit)

1. Find an approximate value for the integral

$$\int_0^{\pi} \sin^3(\phi)\, e^{i\,20\sin(\phi)}$$

using the method of stationary phase. How close is your approximation to the exact value? How could you get a more accurate approximation?

The integral can be evaluated exactly using Mathematica or another symbolic math package. Or, you can use numerical integration with Matlab:

```
a = 0;
b = pi;
N = 1000;
dx = pi/N;
x = (a + dx/2):dx:(b - dx/2);
I = sum(sin(x).^3.*exp(i*20*sin(x)))*dx
```

2. Derive Stirling's formula $n! \simeq (2\pi)^{1/2} n^{n+1/2} e^{-n}$ from the integral definition

$$n! = \int_0^{\infty} x^n e^{-x}\, dx$$

Hint: combine the exponents using the $\ln$ function before finding the critical point. Also, it's helpful to graph the integrand and understand how it depends on n.

3. The Bessel function can be expressed as the integral

$$J_n(\rho) = \mathrm{Re}\left\{ \frac{1}{\pi} \int_0^{\pi} e^{i\rho \sin x} e^{-inx}\, dx \right\}$$

Use the method of stationary phase to derive the asymptotic approximation

$$J_n(\rho) \simeq \sqrt{\frac{2}{\pi\rho}} \cos(\rho - n\pi/2 - \pi/4)$$